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Large Amplitude Free Flexural Vibration of Stiffened Plates

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Large amplitude, free flexural vibration of stiffened plates has been investigated by the spline finite strip method. This is the first attempt to get a satisfactory solution of this problem by spline finite strip method in the form of an eigenvalue problem. The effects of large amplitude have been taken into account by adopting von Kármán's large deflection plate theory and the formulation has been done in the total Lagrangian coordinate system. The resulting nonlinear equations have been solved by the newly developed method, a linearized updated mode with nonlinear time function approximation. The generalized form of spline finite strip method has been used for the analysis of plates of arbitrary shapes. The stiffener has been elegantly modeled so that it can be placed anywhere within the plate strip and it need not follow the nodal lines. The arbitrary orientation and eccentricity of the stiffener have been incorporated in the formulation. Plates and stiffened plated have been analyzed, and the results have been presented and compared with those of other investigators.

Nomenclature

$A_{x'}$	= cross-sectional area of the stiffener
$[B_L]$	= nonlinear strain matrix
$[B_0]$	= linear strain matrix
b	= width of the plate strip
c	= deflection at the plate center
$[D]$	= elasticity matrix
E	= modulus of elasticity
$\{f\}$	= displacement field
G	= modulus of rigidity
h	= distance between two adjacent knots along a nodal line
$I_{x'}$	= second moment of area of stiffener cross section
i, j	= adjacent nodal lines
$\|J\|$	= Jacobian for plate skin
$\|J_S\|$	= Jacobian of the stiffener
$J_{x'}$	= torsional constant of the stiffener
J_0	= polar moment of inertia of stiffener cross section
$[K]$	= stiffness matrix
$[M]$	= mass matrix
M_x, M_y, M_{xy}	= plate bending moments
$M_{x'}$	= stiffener bending moments
$N_x, N_y, \ldots, N_{xy}$	= plate in-plane forces
$N_{x'}$	= stiffener axial force
$N_1, N_2, \ldots, N_4$	= finite element interpolation functions
$[S_L]$	= initial stress matrix (nonlinear)
$S_{x'}$	= first moment of area of the stiffener cross section
$[S_0]$	= initial stress matrix (linear)
$T_{x'}$	= torsional moment of stiffener
t	= thickness of plate skin
u, v, w	= displacement field
x, y, z	= global axis system of the structure
x'	= stiffener direction at a Gauss point
α	= inclination of stiffener at a Gauss point
$\{\delta\}$	= nodal displacement parameters
$\{\varepsilon\}$	= total strain vector (generalized)
$\{\varepsilon_L\}$	= nonlinear strain vector (generalized)
$\{\varepsilon_0\}$	= linear strain vector (generalized)
λ	= stiffener direction after mapping
ν	= Poisson's ratio
ξ, η	= axis system of the mapped domain
ρ	= mass density
$\{\sigma\}$	= stress resultant vector
ϕ	= B_3-spline function
ω, ω_n	= linear and nonlinear frequencies

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Introduction

THE occurrence of stiffened plates is extensive in various engineering constructions. The flexural vibration of these structures is an important aspect of study so that their behavior under dynamic conditions can be well understood. Plates or stiffened plates belong to the category of thin walled structures, and they may undergo large deformations at any instant when they vibrate. At large deflection/amplitude level, the strain–displacement relationship becomes nonlinear, which makes the governing equations as well as the whole problem nonlinear. The solution of such a complex problem can be obtained by the finite element method as it is the most accurate and versatile technique today, but it takes a considerable amount of computer storage and solution time. To get an economic solution with reasonable accuracy, a semianalytical finite strip method[1] has been proposed, particularly for the regular shaped structures, but it suffers from a number of drawbacks such as mixed boundary conditions, continuous span, internal opening, interior supports, and some similar features. The spline finite strip method[2] has recently been proposed as a complement to the semianalytical finite strip method. In this method, spline functions are used instead of characteristic beam functions in the strip direction, which has helped to overcome most of the shortcomings of the conventional finite strip method. Further, the method has been generalized to analyze plates having general shapes.[3] The authors have applied the method to the linear analysis[4,5] of stiffened plates.

The large amplitude vibration of plates has drawn considerable attention of research workers.[6] Herrmann[7] has extended von Kármán's large deflection plate theory to the large amplitude vibration of plates. The simply supported rectangular plate has been investigated by Chu and Herrmann[8] using approximate solution of the nonlinear coupled equations.[7] A first-term approximation in the time series has been used by Yamaki[9] to study the nonlinear vibration of plates having different shapes and boundary conditions. The first attempt to apply the finite element method to large amplitude vibration of plates and beams is due to Mei.[10] His formulation is based on a geometric stiffness matrix where the in-plane forces are due to the large deflection of the structure. Certain approximations have been made to determine these in-plane forces, and these forces are kept constant within an element. Based on the idea of Mei,[10] Rao et al.[11] presented a simple finite element model where they used a linearizing technique for the nonlinear strain–displacement relations.

Prathap and Varadan[12] studied the large amplitude vibration of beams. They presented the formulation of a nonlinear stiffness matrix in their investigation in which the frequencies they obtained are found to be higher than the analytical solution.[12,13] The displacement parameters they used for the evaluation of a nonlinear stiffness matrix have been taken at the instant of maximum amplitude. Actually, the deflection varies from zero to its maximum value and an overhardening effect is obvious with its maximum value. In this context, they attempted to give a new definition of the problem of nonlinear vibration.[12] They used the same time function,[12] i.e., the maximum value of deflection (which actually varies with time) in the Duffing equation formed in the analytical solution,[13] and obtained a set of new results to compare with their finite element results.

Though the results obtained by the methods proposed by Mei[10] and Rao et al.[11] are close to analytical,[13] these methods are not free from errors. There are two flaws in these methods that compensate for each other. The first one is related to formulation of the stiffness matrix, and the second one is the time function, which is similar to that of Prathap and Varadan.[12] Recently, Gray et al.[14] presented a method based on the finite element method where the ambiguities of the earlier methods[10–12] have been overcome. The major difficulty was associated with the time function and this has been taken care of by a proper linearlization of the assumed limit cycle time function.[14] The formulation of the nonlinear stiffness matrix has also been treated properly.[14] In this context, Singh et al.[15] presented a discussion on the different formulations available for the nonlinear vibration of beams.

Though a considerable amount of research has been done on nonlinear vibration of beams and plates, there is a lack of information on large amplitude vibration of stiffened plates.[16–18] Rao et al.[18] used the time function proposed by Prathap and Varadan[12] in their finite element formulation, which is responsible for getting higher values of frequencies. Here nonlinear vibration of stiffened plates is studied by the spline finite strip method after the approach proposed by Gray et al.[14] The formulation is done in the total Lagrangian coordinate system[19] using Herrmann's plate equations.[7]

Proposed Analysis

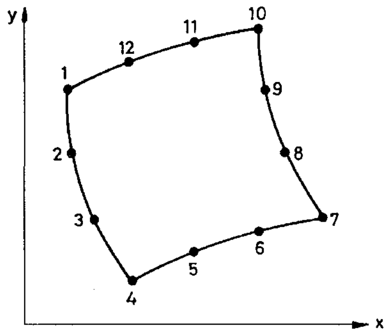
The major assumptions made in the formulation are as follows: the lateral deflection is moderate; the material is linearly elastic; and the normal common to the plate and stiffener system before bending remains straight and normal to the deflected middle plane of the plate after bending.

To cater to the arbitrary plate geometry, the whole plate is approximately mapped into a $[-1, +1]$ region in a different ξ–η plane as shown in Fig. 1 using cubic serendipity shape functions,[19] i.e.,

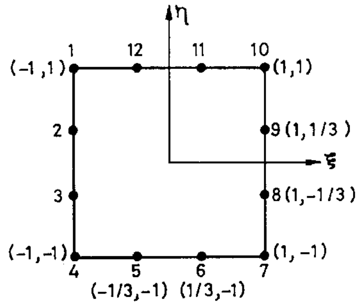
$$x = \sum_{p=1}^{12} N_p(\xi, \eta) x_p$$
$$y = \sum_{p=1}^{12} N_p(\xi, \eta) y_p \tag{1}$$

where N_p is the cubic serendipity shape function and (x_p, y_p) are the coordinates of the ith boundary node of the plate.

In the context of the spline finite strip method, the mapped domain of the plate has been divided into a number of strips in one direction, say, ξ, as shown in Fig. 2. The knots have been taken along the nodal line to adopt the spline function, which serves as the displacement interpolation function in the η direction. The finite element interpolation functions (Hermitian polynomials) are used in the other direction, i.e., the ξ-direction. In this formulation, cubic B splines and cubic finite element interpolation functions have been used for both in-plane and lateral displacements. Thus, the combined interpolation function for any displacement component



a) Original plate shape



b) Mapped shape

Fig. 1 Transformation of an arbitrary plate geometry into a square domain by mapping through cubic serendipity functions.

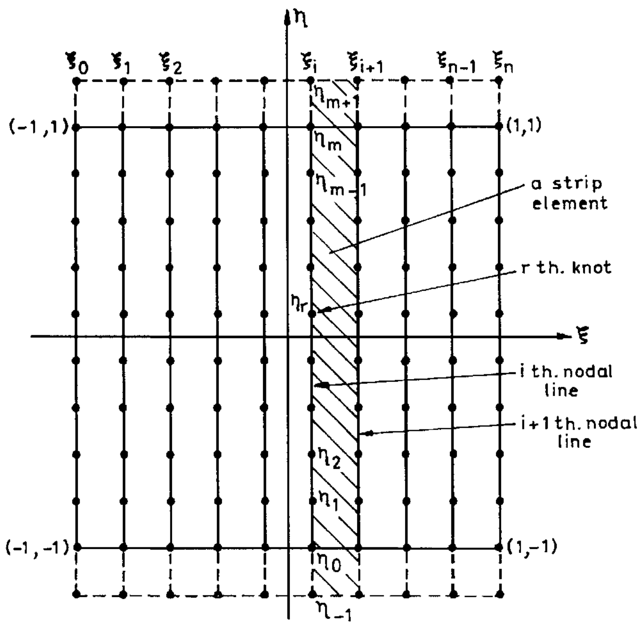


Fig. 2 Mesh division of the mapped domain.

is the product of finite element interpolation functions and spline functions. Now, the displacement components may be expressed as

$$\{f\} = \begin{Bmatrix} u(\xi,\eta) \\ v(\xi,\eta) \\ w(\xi,\eta) \end{Bmatrix} = \begin{bmatrix} [N_{mu}] \\ [N_{mv}] \\ [N_{mw}] \end{bmatrix} \{\delta\} = \begin{bmatrix} [N_u] \\ [N_v] \\ [N_w] \end{bmatrix} [\Phi]\{\delta\} \tag{2}$$

where

$$[N_u] = [N_1(\xi) \;\; N_2(\xi) \;\; 0 \;\; 0 \;\; N_3(\xi) \;\; N_4(\xi) \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$$
$$[N_v] = [0 \;\; 0 \;\; N_1(\xi) \;\; N_2(\xi) \;\; 0 \;\; 0 \;\; N_3(\xi) \;\; N_4(\xi) \;\; 0 \;\; 0 \;\; 0 \;\; 0] \tag{3}$$
$$[N_w] = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; N_1(\xi) \;\; N_2(\xi) \;\; N_3(\xi) \;\; N_4(\xi)]$$

$$
[\Phi] = \begin{bmatrix}
[\phi_u]_i & & & & & & & & & & & \\
& [\phi_{u,\xi}]_i & & & & & & & & & & \\
& & [\phi_v]_i & & & & & & & & & \\
& & & [\phi_{v,\xi}]_i & & & & & 0 & & & \\
& & & & [\phi_u]_{i+1} & & & & & & & \\
& & & & & [\phi_{u,\xi}]_{i+1} & & & & & & \\
& & & & & & [\phi_v]_{i+1} & & & & & \\
& & & & & & & [\phi_{v,\xi}]_{i+1} & & & & \\
& & & & & & & & [\phi_w]_i & & & \\
& & & 0 & & & & & & [\phi_{w,\xi}]_i & & \\
& & & & & & & & & & [\phi_w]_{i+1} & \\
& & & & & & & & & & & [\phi_{w,\xi}]_{i+1}
\end{bmatrix} \tag{4}
$$

and

$$
\{\delta\}^T = \left[\{u\}_i^T \quad \{u_{,\xi}\}_i^T \quad \{v\}_i^T \quad \{v_{,\xi}\}_i^T \quad \{u\}_{i+1}^T \quad \{u_{,\xi}\}_{i+1}^T \quad \{v\}_{i+1}^T \quad \{v_{,\xi}\}_{i+1}^T \quad \{w\}_i^T \quad \{w_{,\xi}\}_i^T \quad \{w\}_{i+1}^T \quad \{w_{,\xi}\}_{i+1}^T\right] \tag{5}
$$

Again, $[\phi]_i = [\phi_{-1}\ \phi_0\ \phi_1\ \phi_2\ \cdots\ \phi_r\ \cdots\ \phi_m\ \phi_{m+1}]_i$ (the spline functions adopted along the ith nodal line for a displacement component) and $\{u\}_i^T = \{u_{-1}\ u_0\ u_1\ u_2\ \cdots\ u_r\ \cdots\ u_m\ u_{m+1}\}_i$ (the displacement parameters at the ith nodal line for u). The expressions for the polynomials of cubic-B splines at any knot, say, rth point (ϕ_r), are as follows:

$$
\phi_r = \frac{1}{6h^3}\begin{Bmatrix}
(\eta - \eta_{r-2})^3, & \eta_{r-2} \le \eta \le \eta_{r-1} \\
h^3 + 3h^2(\eta - \eta_{r-1}) + 3h(\eta - \eta_{r-1})^2 - 3(\eta - \eta_{r-1})^3, & \eta_{r-1} \le \eta \le \eta_r \\
h^3 + 3h^2(\eta_{r+1} - \eta) + 3h(\eta_{r+1} - \eta)^2 - 3(\eta_{r+1} - \eta)^3, & \eta_r \le \eta \le \eta_{r+1} \\
(\eta_{r+2} - \eta)^3, & \eta_{r+1} \le \eta \le \eta_{r+2}
\end{Bmatrix} \tag{6}
$$

The finite element interpolation functions $N_1(\xi)$–$N_4(\xi)$ are the Hermitian polynomials of first order used in a standard beam element.

Once the interpolation functions for the displacements are obtained, the stiffness matrix and mass matrix can be formed in a manner similar to that followed in finite element method. The formulation has been done in total Lagrangian coordinate system following the approach of Zienkiewicz.[19–21] In a general form, the stiffness matrix of an element may be written as

$$
\begin{aligned}
[K_e] = &\int_v [B_0]^T[D][B_0]\,\mathrm{d}v + \frac{1}{2}\int_v [B_0]^T[D][B_L]\,\mathrm{d}v \\
&+ \frac{1}{2}\int_v [B_L]^T[D][B_0]\,\mathrm{d}v + \frac{1}{3}\int_v [B_L]^T[D][B_L]\,\mathrm{d}v \\
&+ \frac{1}{2}\int_v [G]^T[S_0][G]\,\mathrm{d}v + \frac{1}{3}\int_v [G]^T[S_L][G]\,\mathrm{d}A
\end{aligned} \tag{7}
$$

(See the Nomenclature and the stiffness matrices of the plate strip and the stiffener for the different matrices used in the preceding equation.)

The stiffness matrices of the plate strips/elements and those of the stiffener elements have been assembled together, which gives the overall stiffness matrix of the structure $[K]$. In the similar manner, the overall mass matrix $[M]$ has been generated by assembling the element mass matrices. In the context of the present problem, the overall system equation may be written as

$$
[K]\{\delta\} + [M]\{\ddot{\delta}\} = \{0\} \tag{8}
$$

Now the kinematic boundary conditions have been imposed in the preceding equation. This has been attained through proper amendment of the local splines near the two ends of the nodal lines for the boundaries/sides perpendicular to nodal lines ($\eta = +1$ and -1) (Ref. 3). The technique followed in the finite element method has been adopted for the other sides ($\xi = +1$ and -1).

Equation (8) has been solved by the technique proposed by Gray et al.,[14] which consists of using a linearized updated mode with a nonlinear time function approximation (LUM/NTF). Following the approach of Gray et al.,[14] the system of equations has been reduced to a system containing the transverse displacements only. Now, the problem is with the evaluation of the nonlinear stiffness parameters as these quantities are dependent on time varying displacements. The assumed time function for the displacements has been approximated to a quantity that is independent of time. This has been done by neglecting the higher harmonic terms. With the help of these approximations, the reduced system equation has been linearized, and it has been solved in a iterative manner.

The different matrices mentioned in Eq. (7) are presented in the following sections corresponding to the plate strip and stiffener element.

Stiffness Matrix of the Plate Strip

The formulation for the plate strip is identical to that of a bare plate, which is well documented elsewhere.[19–21] Thus, the matrices corresponding to a plate strip are presented here in the final forms. The generalized rigidity matrix is written as

$$
[D] = \frac{Et}{1-\nu^2}\begin{bmatrix}
1 & \nu & & & & \\
\nu & 1 & & & & 0 \\
& & (1-\nu)/2 & & & \\
& & & t^2/12 & \nu t^2/12 & \\
& 0 & & \nu t^2/12 & t^2/12 & \\
& & & & & \dfrac{(1-\nu)t^2}{24}
\end{bmatrix} \tag{9}
$$

With the help of displacement interpolation functions (2), the linear strain matrix is expressed as

$$
[B_0] = \begin{bmatrix}
[N_{mu}]_{,x} \\
[N_{mv}]_{,y} \\
[N_{mu}]_{,y} + [N_{mv}]_{,x} \\
-[N_{mw}]_{,xx} \\
-[N_{mw}]_{,yy} \\
-2[N_{mw}]_{,xy}
\end{bmatrix} \tag{10}
$$

The nonlinear strain matrix may be expressed as

$$[B_L] = [A][G] \tag{11}$$

where

$$[A] = \begin{bmatrix} [N_{mw}]_{,x}\{\delta\} & 0 \\ 0 & [N_{mw}]_{,y}\{\delta\} \\ [N_{mw}]_{,y}\{\delta\} & [N_{mw}]_{,x}\{\delta\} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{12}$$

and

$$[G] = \begin{bmatrix} [N_{mw}]_{,x} \\ [N_{mw}]_{,y} \end{bmatrix} \tag{13}$$

These equations indicate that $[A]$ is dependent on displacements but $[G]$ is free from them.

The matrices $[S_0]$ and $[S_L]$ obtained in Eq. (7) contain the contributions of membrane forces corresponding to linear and nonlinear strain terms, respectively. These matrices may be expressed as follows:

$$[S_0] = \begin{bmatrix} N_x^0 & N_{xy}^0 \\ N_{xy}^0 & N_y^0 \end{bmatrix} \tag{14}$$

and

$$[S_L] = \begin{bmatrix} N_x^L & N_{xy}^L \\ N_{xy}^L & N_y^L \end{bmatrix} \tag{15}$$

where

$$\begin{aligned} N_x^0 &= Et/(1-\nu^2)([N_{mu}]_{,x}\{\delta\} + \nu[N_{mv}]_{,y}\{\delta\}) \\ N_y^0 &= Et/(1-\nu^2)([N_{mv}]_{,y}\{\delta\} + \nu[N_{mu}]_{,x}\{\delta\}) \\ N_{xy}^0 &= Et/\{2(1+\nu)\}([N_{mu}]_{,y}\{\delta\} + [N_{mv}]_{,x}\{\delta\}) \\ N_x^L &= Et/\{2(1-\nu^2)\}\left\{([N_{mw}]_{,x}\{\delta\})^2 + \nu([N_{mw}]_{,y}\{\delta\})^2\right\} \\ N_y^L &= Et/\{2(1-\nu^2)\}\left\{([N_{mw}]_{,y}\{\delta\})^2 + \nu([N_{mw}]_{,x}\{\delta\})^2\right\} \\ N_{xy}^L &= Et/\{2(1+\nu)\}\{([N_{mw}]_{,x}\{\delta\})([N_{mw}]_{,y}\{\delta\})\} \end{aligned} \tag{16}$$

Now, all of the terms of Eq. (7) can be expressed in terms of ξ and η. Thus, the stiffness matrix of a plate strip may be expressed in a compact form as

$$[K_e] = \int f(\xi,\eta)\,\mathrm{d}x\,\mathrm{d}y = \int f(\xi,\eta)|J|\,\mathrm{d}\xi\,\mathrm{d}\eta \tag{17}$$

where

$$|J| = \begin{vmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \eta} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \xi} \end{vmatrix} \tag{18}$$

The Gauss quadrature method has been used to carry out the integrations involved in Eq. (17) numerically.

Stiffness Matrix of the Stiffener

An attempt has been made to model an eccentric stiffener that may be oriented to any direction and that may lie anywhere within the plate strip, following the concept of Mukherjee and Mukhopadhyay.[22] The basic idea lies with the use of the same interpolation functions for the plate and the stiffener displacements. This not only ensures compatibility between the plate and the stiffener but also avoids incorporation of additional degrees of freedom for the stiffener element. As the stiffener orientation may not follow

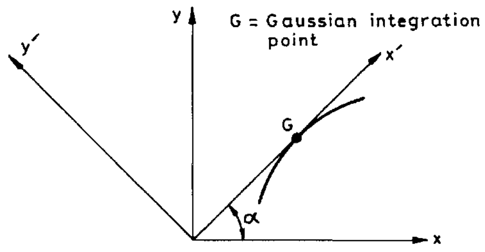


Fig. 3 Axes system at a Gaussian integration point of a stiffener.

the structural axis system and it may vary from point to point (for a curved stiffener), it is necessary to transform the stiffener parameters accordingly. The stiffener parameters have been taken at the point of evaluation (Gaussian integration point) along the tangential direction x' (Fig. 3). This gives the result in a local axis system (x'–y') that is required to convert into the structural axis system (x–y). Let α be the angle made by the local axis system at the Gauss point with the structural axis system (Fig. 3).

The structural axis system x–y and the local axis system x'–y' can be related as follows:

$$\begin{aligned} x &= x'\cos\alpha - y'\sin\alpha \\ y &= x'\sin\alpha + y'\cos\alpha \end{aligned} \tag{19}$$

The local displacement components can be expressed in terms of global displacement components as

$$\begin{aligned} u' &= u\cos\alpha + v\sin\alpha \\ v' &= v\cos\alpha - u\sin\alpha \\ w' &= w \end{aligned} \tag{20}$$

The stress resultant vector of the stiffener in its local axis system may be written as

$$\{\sigma\}^T = \{\,N_{x'} \quad M_{x'} \quad T_{x'}\,\} \tag{21}$$

The corresponding rigidity matrix may be expressed as

$$[D] = \begin{bmatrix} EA_{x'} & ES_{x'} & 0 \\ ES_{x'} & EI_{x'} & 0 \\ 0 & 0 & GJ_{x'} \end{bmatrix} \tag{22}$$

For a stiffener having eccentricity with respect to the reference plane, $S_{x'}$ becomes nonzero, which provides the coupling between the membrane action and the flexural action. The coupling will also be obtained because of large deformations, and it will be reflected in the strain vector, which is as follows:

$$\{\varepsilon\} = \{\varepsilon_0\} + \{\varepsilon_L\} = \begin{Bmatrix} \dfrac{\partial u'}{\partial x'} \\ \text{- - - - - -} \\ -\dfrac{\partial^2 w}{\partial x'^2} \\ -\dfrac{\partial^2 w}{\partial x' \partial y'} \end{Bmatrix} + \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x'}\right)^2 \\ \text{- - - - - -} \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

With the help of Eq. (20) and the interpolation functions (2), the strain vectors may be expressed in terms of nodal parameters as follows.

The linear strain vector may be written as

$$\{\varepsilon_0\} = [B_0]\{\delta\} \tag{24}$$

where

$$[B_0] = \begin{bmatrix} [N_{mu}]_{,x'}\cos\alpha + [N_{mv}]_{,x'}\sin\alpha \\ -[N_{mw}]_{,x'x'} \\ -[N_{mw}]_{,x'y'} \end{bmatrix} \tag{25}$$

The nonlinear strain vector may be expressed as

$$\{\varepsilon_L\} = \frac{1}{2}\begin{bmatrix} \dfrac{\partial w}{\partial x'} \\ 0 \\ 0 \end{bmatrix}\left\{\frac{\partial w}{\partial x'}\right\} = \frac{1}{2}[A]\{\theta\} = \frac{1}{2}[A][G]\{\delta\} \tag{26}$$

where

$$[A] = \begin{bmatrix} [N_{mw}]_{,x'}\{\delta\} \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

and

$$[G] = [N_{mw}]_{,x'} \tag{28}$$

For the stiffener, the matrices $[S_0]$ and $[S_L]$ will be as follows:

$$[S_0] = EA_{x'}([N_{mu}]_{,x'}\{\delta\}\cos\alpha + [N_{mv}]_{,x'}\{\delta\}\sin\alpha) - ES_{x'}[N_{mw}]_{,x'}\{\delta\} \tag{29}$$

$$[S_L] = \frac{EA_{x'}([N_{mw}]_{,x'}\{\delta\})^2}{2} \tag{30}$$

The derivatives with respect to x' and y' can be transformed into x and y with the help of Eq. (19), which relates the local and global axis system.

Here, the integrations involved in the evaluation of the stiffness matrix given by Eq. (7) should be carried out along the axis of the stiffener. Thus, the stiffness matrix of the stiffener may be written as

$$[K_e] = \int_{x'} f(\xi,\eta)\,dx' = \int_{\lambda} f(\xi,\eta)|J_S|\,d\lambda \tag{31}$$

Mass Matrix of the Plate Strip and the Stiffener

A consistent mass matrix has been formulated. The mass contributions corresponding to the bending displacements only have been taken into account. The mass moment of inertia has been included in the formulation. As the effect of mass components corresponding to the in-plane displacements is negligible particularly in the lower modes, their contributions have not been included in the mass matrix.

Using the interpolation functions corresponding to bending displacements (2), the mass matrix of the plate strip may be written as

$$[M_e] = \rho t\int_v \left\{[N_{mw}]^T[N_{mw}] + \left(\frac{t^2}{12}\right)([N_{mw}]_{,x}^T[N_{mw}]_{,x} + [N_{mw}]_{,y}^T[N_{mw}]_{,y})\right\}|J|\,d\xi\,d\eta \tag{32}$$

After performing the necessary transformations of the stiffener parameters from the local to the global axis system, the mass matrix of the stiffener may be written as

$$[M_e] = \rho\int_{\lambda}\Big\{A_{x'}[N_{mw}]^T[N_{mw}] + I_{x'}([N_{mw}]_{,x}\cos\alpha + [N_{mw}]_{,y}\sin\alpha)^T([N_{mw}]_{,x}\cos\alpha + [N_{mw}]_{,y}\sin\alpha) + J_0([N_{mw}]_{,x}\sin\alpha - [N_{mw}]_{,y}\cos\alpha)^T \times ([N_{mw}]_{,x}\sin\alpha - [N_{mw}]_{,y}\cos\alpha)\Big\}|J_{x'}|\,d\lambda \tag{33}$$

Numerical Examples

As there are no satisfactory results available for nonlinear vibration of stiffened plates, an example of a bare plate has been investigated for the validation of the spline finite strip method. Stiffened plates of different types have been analyzed, and the results obtained are presented along with those of earlier investigators for necessary discussion. The results cannot be compared because the basis of the earlier investigation is different from the present method. The convergence of results with respect to mesh size has been checked for all of the problems. The iteration involved in the solution process has been continued until convergence is attained both for the mode shape and the frequency. The results are presented in the form of nonlinear frequency ratios (ω_n/ω) for different amplitude levels (c/t). For all of the examples presented, the in-plane boundary conditions are $u = v = 0$ at all four edges.

Unstiffened Square Plate

A square plate having all edges simply supported has been analyzed by the proposed method. The results obtained are presented along with the analytical results of Chu and Herrmann[8] and the finite element results of Ganapati et al.[23] and Rao et al.[18] in Table 1. The results correspond to a mesh size of 10 × 8 (10 strips and 8 divisions per strip). Table 1 shows that the results obtained by the proposed method have excellent agreement with the analytical results,[8] which indicates the accuracy of the proposed method. The frequency ratios obtained by Ganapati et al.[23] and Rao et al.[18] are found to be higher than the analytical result,[8] and this is as expected because they used the displacement at the instant of its maximum amplitude for the evaluation of nonlinear stiffness parameters that introduces an overhardening effect.

Again, the plate has been analyzed for a different boundary condition, that is, clamped along its four edges. The results obtained by the proposed method with a mesh size of 10 × 8 are presented along with the analytical results of Yamaki[9] and the finite element results of Rao et al.[18] in Table 2. Similar to the earlier case, it is found that there is an excellent agreement between the present results and the analytical results.[9] The overhardening effect of the finite element formulation of Rao et al.[18] has been reflected again in Table 2.

Rectangular Plate with a Central Eccentric Stiffener

A simply supported rectangular plate stiffened with a central eccentric stiffener along its short span (Fig. 4) has been studied. The nonlinear frequency ratios (ω_n/ω) obtained by the proposed method are presented with the finite element results of Rao et al.[18] in Table 3. The left-hand half of the plate has been analyzed and the results presented in Table 3 correspond to a mesh size of 8 × 8. Table 3 shows that the results of Rao et al.[18] are on the higher side as was expected.

Cross-Stiffened Plate

A simply supported square plate stiffened along both the directions by central eccentric stiffeners (Fig. 5) has been investigated.

Table 1 Nonlinear frequency ratios (ω_n/ω) of a simply supported square plate

c/t	0.2	0.4	0.6	0.8	1.0
Present analysis	1.01970	1.07681	1.16624	1.28132	1.41729
Analytical[8]	1.01947	1.07561	1.16252	1.27339	1.40232
Ganapati et al.[23]	1.02504	1.10021	1.20803	1.35074	1.51347
Rao et al.[18]	1.0261	1.1009	1.2162	1.3624	1.5314

Table 2 Nonlinear frequency ratios (ω_n/ω) of a clamped square plate

c/t	0.2	0.4	0.6	0.8	1.0
Present analysis	1.00726	1.02866	1.06335	1.11001	1.16715
Analytical[9]	1.00847	1.02923	1.06609	1.11358	1.16740
Rao et al.[18]	1.0095	1.0375	1.0825	1.1424	1.2149

Table 3 Nonlinear frequency ratios (ω_n/ω) of a rectangular plate with a central stiffener

c/t	0.2	0.4	0.6	0.8	1.0
Present analysis	1.01467	1.05236	1.10902	1.17947	1.26016
Rao et al.[18]	1.02004	1.07105	1.14653	1.23940	1.34433

Table 4 Nonlinear frequency ratios (ω_n/ω) of a cross stiffened square plate

c/t	0.2	0.4	0.6	0.8	1.0
Present analysis	1.00478	1.01762	1.03705	1.06145	1.08959
Rao et al.[18]	1.0087	1.0269	1.0540	1.0889	1.1310

Table 5 Nonlinear frequency ratios (ω_n/ω) of a stiffened skew plate

c/t	0.2	0.4	0.6	0.8	1.0
Present analysis	1.00153	1.00581	1.01405	1.02474	1.03696
Rao et al.[18]	1.00157	1.00679	1.01562	1.02797	1.04379

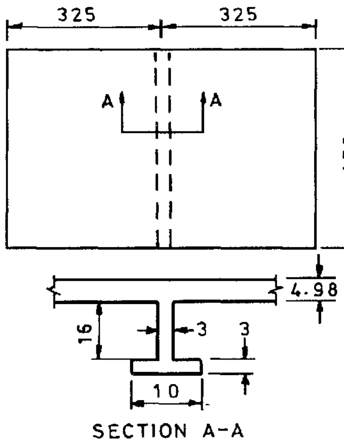


Fig. 4 Rectangular plate having a central stiffener. All dimensions are in millimeters.

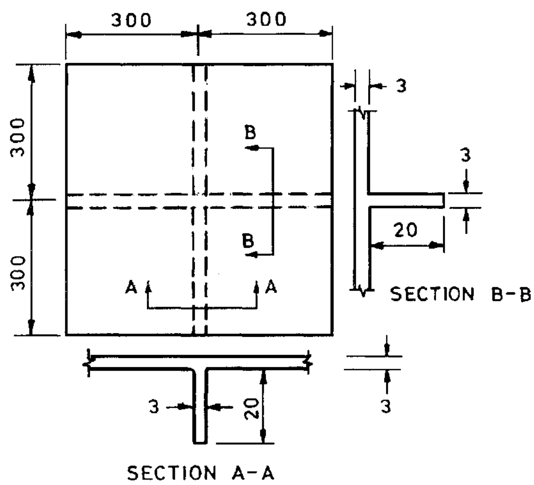


Fig. 5 Cross stiffened plate. All dimensions are in millimeters.

The results obtained by the proposed method are presented with those of Rao et al.[18] in Table 4. The left-hand half of the plate has been analyzed and the results presented in Table 4 correspond to a mesh size of 6 × 10. As expected, the finite element results of Rao et al.[18] are higher than the present results.

Stiffened Skew Plate

A stiffened skew plate having an included angle of 60 deg, as shown in Fig. 6, has been studied. The boundaries of the plate are clamped, and the stiffener properties are identical with those used in the third example. The results obtained by the proposed method for a mesh division of 12 × 8 are presented with the finite element results of Rao et al.[18] in Table 5. Similar to the earlier examples, the finite element results of Rao et al. are found to be higher than the present results.

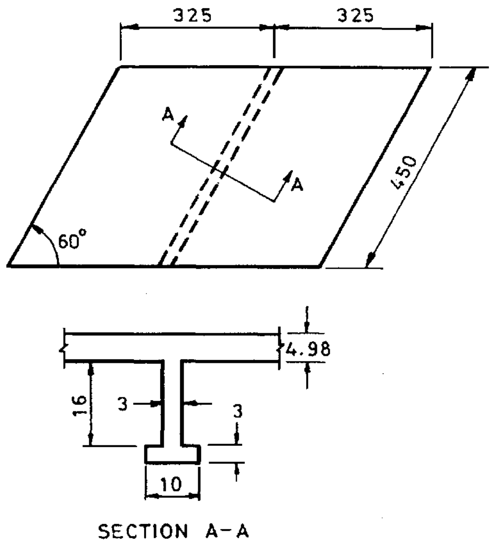


Fig. 6 Skew plate having an eccentric stiffener. All dimensions are in millimeters.

Conclusions

The spline finite strip method has been applied to the problem of nonlinear vibration of stiffened plates. The formulation has been done in such a way that the plate having any shape and the stiffener having any orientation and eccentricity can be analyzed. Again, the stiffener may lie anywhere within the plate strip, which helps to choose any mesh division irrespective of stiffener location. The formulation has been done in the total Lagrangian coordinate system using the dynamic analog of von Kármán's nonlinear field theory. The governing nonlinear equation has been solved by the direct iteration technique following the LUM/NTF method. The proposed method has been validated by solving a problem of a bare plate, which indicates the accuracy of the method. Examples of stiffened plates have been carried out by the proposed method, and the results obtained have been presented in the form of frequency ratio (ω_n/ω) for different amplitude levels (c/t). These results may help future investigations of work in this area.

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